

# Probabilities Are All You Need: A Probability-Only Approach to Uncertainty Estimation in Large Language Models

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**Core idea:** Approximate **Predictive Entropy** using only the response's **top-K probabilities**, where K is adaptively chosen by **thresholding a**

**Question:** What is the fastest animal on Earth?

The fastest animal on Earth is the sloth.



Large Language Model

low uncertainty

high uncertainty

## Abstract

- Large Language Models (LLMs) perform well across NLP tasks but are prone to **hallucinations**—factually incorrect outputs that undermine reliability in real-world applications.
- Estimating uncertainty is a key strategy to detect hallucinations. However, existing methods often require sampling or extra computation to assess predictive entropy.
- We propose a **training-free, efficient method to estimate uncertainty** based on top-K output probabilities.

## Background

In the context of LLMs, we can measure the uncertainty of a generation as:

$$U(x) = H(Y|x) = - \sum_y p(y|x) \log p(y|x)$$

The probability of generating sequence  $y$  given a prompt  $x$ :

$$p(y|x) = \prod_{t=1}^T p(y^t|y^{<t}, x)$$

where  $T$  is the length of the generated sequence, and  $y^t$  is the token at position  $t$ . Taking the logarithm, we get **Negative Log-Likelihood** (NLL):

$$\text{NLL}(y|x) = - \sum_{t=1}^T \log p(y^t|y^{<t}, x).$$

$$p(y|x) = e^{-\text{NLL}(y|x)}$$

However, NLL is **relying solely on a single generation** that can miss plausible alternatives, limiting the ability to capture response uncertainty in ambiguous or high-variance prompts.

## Method

For simplicity, we use  $p_i^*$  to represent the probability of the top  $i$ -th generation  $p(y_i^*|x)$ . We introduce an approximation of **Predictive Entropy** as a **PRobability-Only** uncertainty score (PRO):

$$\text{PRO}(x) = - \log p_K^* - \sum_{i=1}^K p_i^* \log \frac{p_i^*}{p_K^*}$$

**Proposition 1.** Let  $\mathbf{y}^* = (y_1^*, y_2^*, \dots, y_K^*)$  be the top  $K$  generations of a LLM given prompt  $x$ . The predictive entropy approximation using the top  $K$  probabilities satisfies the following inequality:

$$H(Y|x) \geq - \log p_K^* - \sum_{i=1}^K p_i^* \log \frac{p_i^*}{p_K^*}$$

## Adaptive Top-K selection

Instead of using a fixed top-K, we propose an **adaptive constraint** that filters out low-probability generations, ensuring the uncertainty estimation focuses on the most confident and relevant responses.

$$\mathbf{p}_K = \{p_k \mid p_k \geq \alpha, 1 \leq k \leq N\}$$

## Experiments

- Baselines:** Semantic-based methods (SD, SE, Deg), Predictive Entropy (NE, PE), Negative Log-likelihood (ALL, NLL)

Dataset	Model	SD	SE	Deg	NE	PE	ALL	NLL	PRO (Ours)
TriviaQA	Gemma-2B	0.799	0.668	0.746	0.692	0.624	0.789	0.806	<b>0.819</b>
	Gemma-7B	0.831	0.690	0.715	0.702	0.652	<u>0.833</u>	0.812	<b>0.841</b>
	Llama2-13B	<b>0.862</b>	0.682	<u>0.802</u>	0.551	0.552	0.624	0.684	<b>0.802</b>
	Falcon-11B	<u>0.706</u>	0.592	<b>0.710</b>	0.555	0.604	0.577	0.668	0.668
	Falcon-40B	0.700	<u>0.724</u>	0.722	0.674	0.623	0.658	<b>0.765</b>	<b>0.765</b>
SciQ	Gemma-2B	0.719	0.570	0.725	0.601	0.605	0.719	<u>0.728</u>	<b>0.751</b>
	Gemma-7B	0.741	0.622	0.699	0.658	0.678	<u>0.765</u>	0.755	<b>0.787</b>
	Llama2-13B	0.706	0.574	<b>0.720</b>	0.481	0.543	0.515	0.600	<b>0.716</b>
	Falcon-11B	0.724	0.554	0.771	0.561	0.603	0.573	<u>0.797</u>	<b>0.799</b>
	Falcon-40B	<u>0.668</u>	0.613	0.626	0.592	0.577	0.660	<b>0.674</b>	<b>0.674</b>
NQ	Gemma-2B	0.618	0.599	0.620	0.600	0.613	0.607	<u>0.694</u>	<b>0.696</b>
	Gemma-7B	0.670	0.621	<u>0.691</u>	0.662	0.566	<b>0.698</b>	0.683	<b>0.691</b>
	Llama2-13B	0.627	0.562	0.713	0.540	0.649	0.691	<u>0.737</u>	<b>0.740</b>
	Falcon-11B	0.636	0.591	0.580	0.515	0.522	0.512	0.684	<b>0.685</b>
	Falcon-40B	0.632	0.603	0.579	0.544	0.585	0.475	<u>0.638</u>	<b>0.645</b>
Average AUC		0.709	0.618	0.695	0.595	0.600	0.646	0.715	<b>0.739</b>
Best Count		1	0	2	0	0	1	2	<b>11</b>

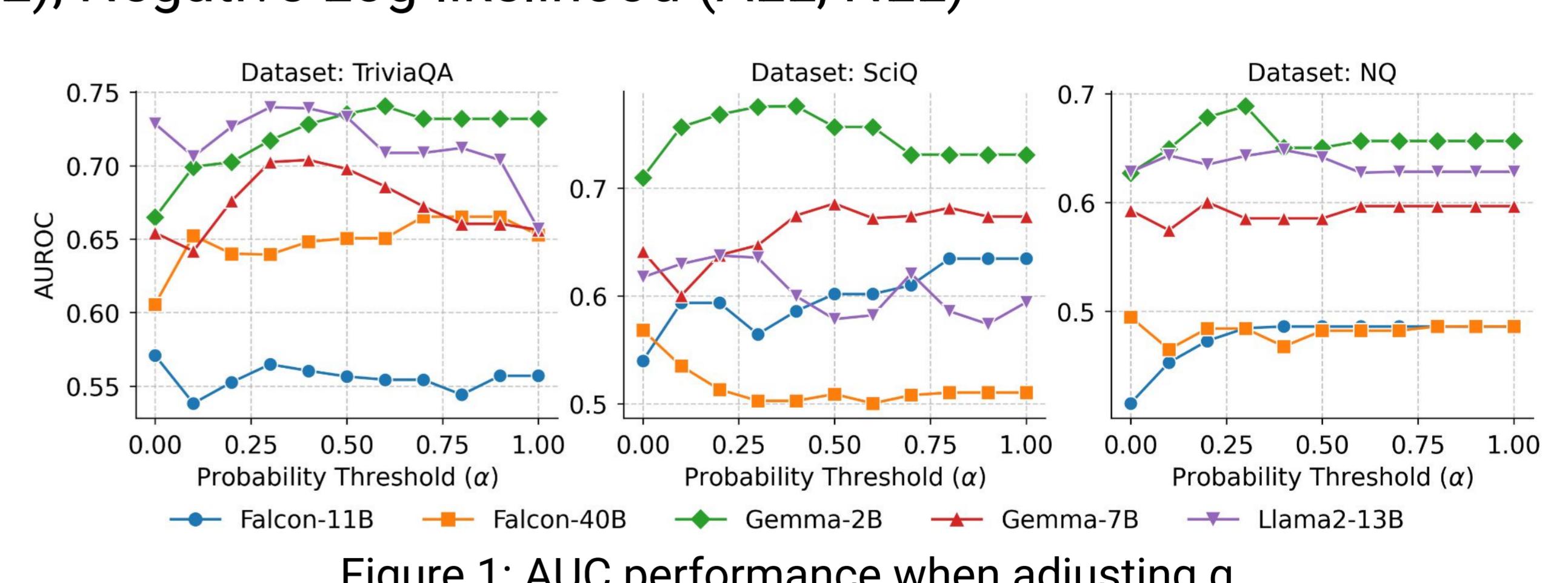


Figure 1: AUC performance when adjusting  $\alpha$

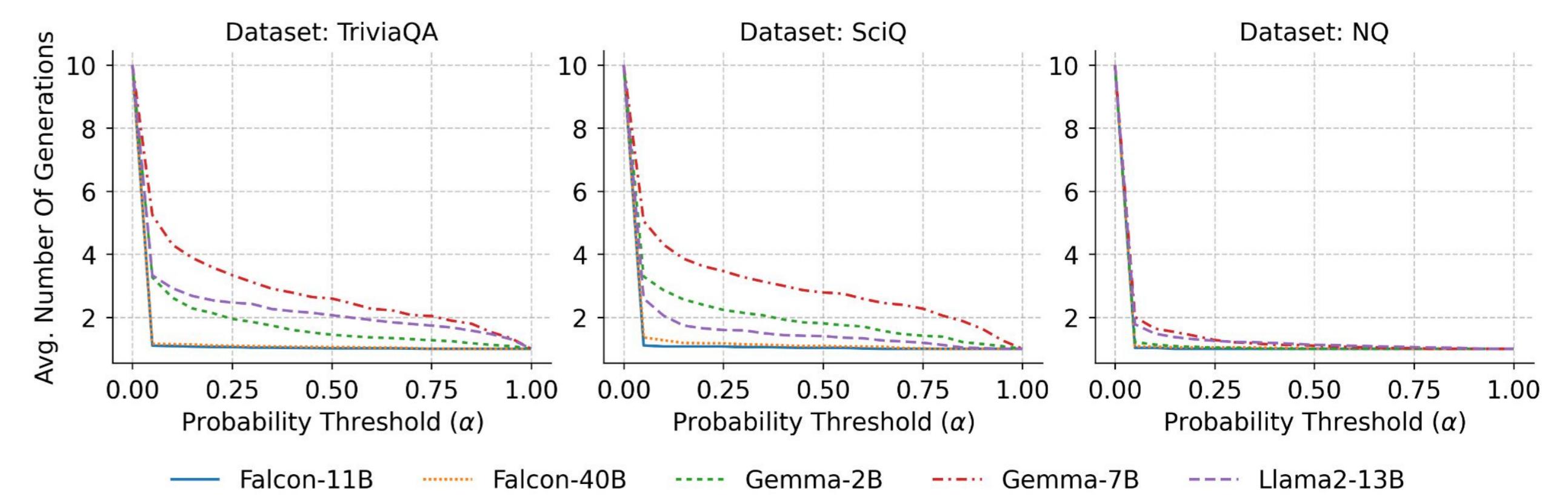


Figure 2: Relationship between number of selected generations and  $\alpha$