

Probabilities Are All You Need: A Probability-Only Approach to Uncertainty Estimation in Large Language Models

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Core idea: Approximate **Predictive Entropy** using only the response's **top-K probabilities**, where K is adaptively chosen by **thresholding α**

Question: What is the fastest animal on Earth?

The fastest animal on Earth is the sloth.



Large Language Model

low uncertainty

high uncertainty

Background

In the context of LLMs, we can measure the uncertainty of a generation as:

$$U(x) = H(Y|x) = - \sum_y p(y|x) \log p(y|x)$$

The probability of generating sequence y given a prompt x :

$$p(y|x) = \prod_{t=1}^T p(y^t | y^{<t}, x)$$

where T is the length of the generated sequence, and y^t is the token at position t . Taking the logarithm, we get **Negative Log-Likelihood** (NLL):

$$\text{NLL}(y|x) = - \sum_{t=1}^T \log p(y^t | y^{<t}, x).$$

$$p(y|x) = e^{-\text{NLL}(y|x)}$$

However, NLL is **relying solely on a single generation** that can miss plausible alternatives, limiting the ability to capture response uncertainty in ambiguous or high-variance prompts.

Abstract

- Large Language Models (LLMs) perform well across NLP tasks but are prone to **hallucinations**—factually incorrect outputs that undermine reliability in real-world applications.
- Estimating uncertainty is a key strategy to detect hallucinations. However, existing methods often require sampling or extra computation to assess predictive entropy.
- We propose a **training-free, efficient method to estimate uncertainty** based on top-K output probabilities.

Method

For simplicity, we use p_i^* to represent the probability of the top i -th generation $p(y_i^* | x)$. We introduce an approximation of **Predictive Entropy** as a **PRobability-Only** uncertainty score (PRO):

$$\text{PRO}(x) = -\log p_K^* - \sum_{i=1}^K p_i^* \log \frac{p_i^*}{p_K^*}$$

Proposition 1. Let $\mathbf{y}^* = (y_1^*, y_2^*, \dots, y_K^*)$ be the top K generations of a LLM given prompt x . The predictive entropy approximation using the top K probabilities satisfies the following inequality:

$$H(Y|x) \geq -\log p_K^* - \sum_{i=1}^K p_i^* \log \frac{p_i^*}{p_K^*}$$

Adaptive Top-K selection

Instead of using a fixed top-K, we propose an **adaptive constraint** that filters out low-probability generations, ensuring the uncertainty estimation focuses on the most confident and relevant responses.

$$\mathbf{p}_K = \{p_k \mid p_k \geq \alpha, 1 \leq k \leq N\}$$

Experiments

- Baselines:** Semantic-based methods (SD, SE, Deg), Predictive Entropy (NE, PE), Negative Log-likelihood (ALL, NLL)

Dataset	Model	SD	SE	Deg	NE	PE	ALL	NLL	PRO (Ours)
TriviaQA	Gemma-2B	0.799	0.668	0.746	0.692	0.624	0.789	0.806	0.819
	Gemma-7B	0.831	0.690	0.715	0.702	0.652	0.833	0.812	0.841
	Llama2-13B	0.862	0.682	0.802	0.551	0.552	0.624	0.684	0.802
	Falcon-11B	0.706	0.592	0.710	0.555	0.604	0.577	0.668	0.668
	Falcon-40B	0.700	0.724	0.722	0.674	0.623	0.658	0.765	0.765
SciQ	Gemma-2B	0.719	0.570	0.725	0.601	0.605	0.719	0.728	0.751
	Gemma-7B	0.741	0.622	0.699	0.658	0.678	0.765	0.755	0.787
	Llama2-13B	0.706	0.574	0.720	0.481	0.543	0.515	0.600	0.716
	Falcon-11B	0.724	0.554	0.771	0.561	0.603	0.573	0.797	0.799
	Falcon-40B	0.668	0.613	0.626	0.592	0.577	0.660	0.674	0.674
NQ	Gemma-2B	0.618	0.599	0.620	0.600	0.613	0.607	0.694	0.696
	Gemma-7B	0.670	0.621	0.691	0.662	0.566	0.698	0.683	0.691
	Llama2-13B	0.627	0.562	0.713	0.540	0.649	0.691	0.737	0.740
	Falcon-11B	0.636	0.591	0.580	0.515	0.522	0.512	0.684	0.685
	Falcon-40B	0.632	0.603	0.579	0.544	0.585	0.475	0.638	0.645
Average AUC		0.709	0.618	0.695	0.595	0.600	0.646	0.715	0.739
Best Count		1	0	2	0	0	1	2	11

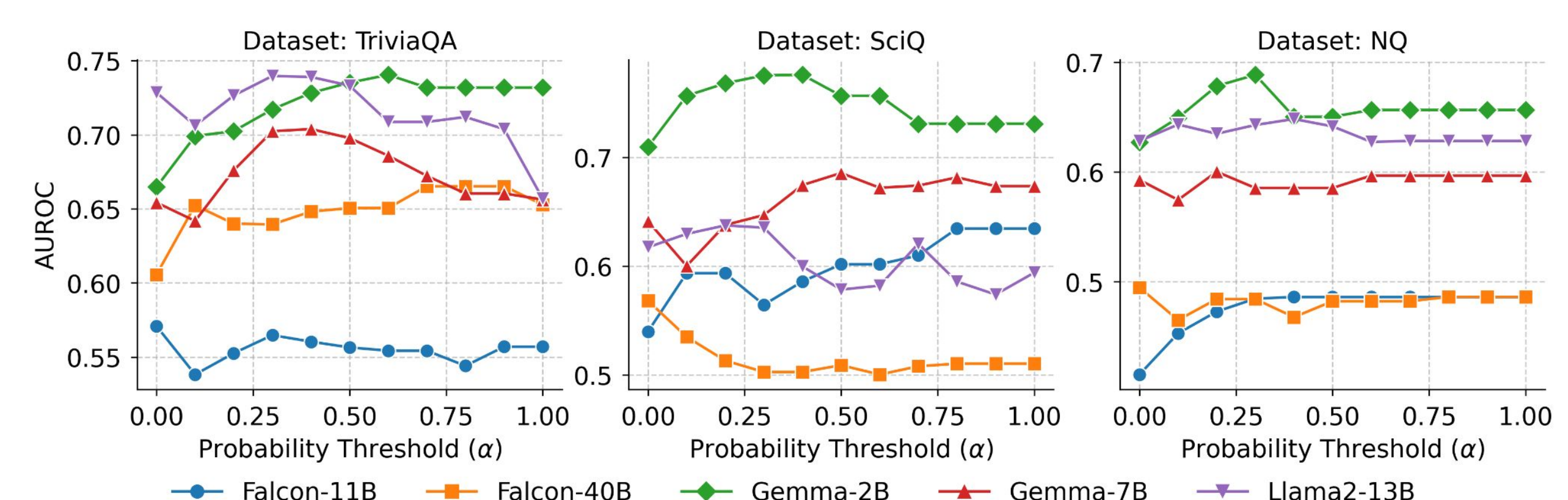


Figure 1: AUC performance when adjusting α

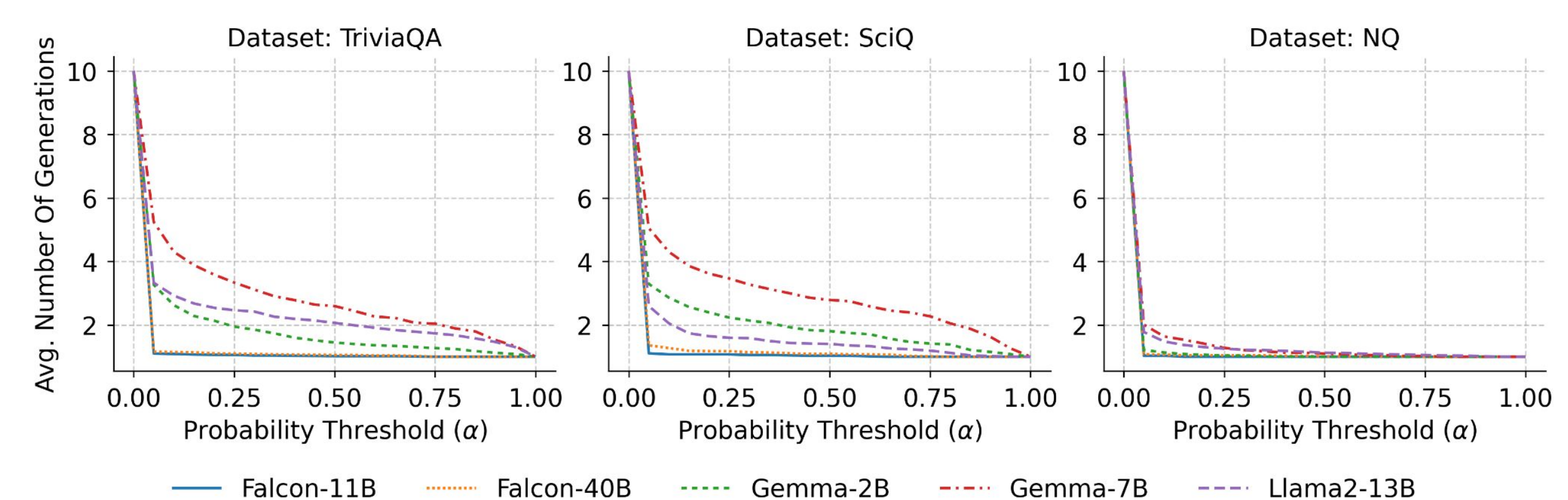


Figure 2: Relationship between number of selected generations and α